Chapter 17 APPENDIX A

This appendix shows one method for computation of the PSD for a BOC(n,m) modulation spreading symbol with p even (=2n/m). An alternative is through the path of deriving the ACF for the convolution of the spreading symbol with the ideal code sequence and using the Weiner-Khinchine Theorem to convert from the time to frequency domains.

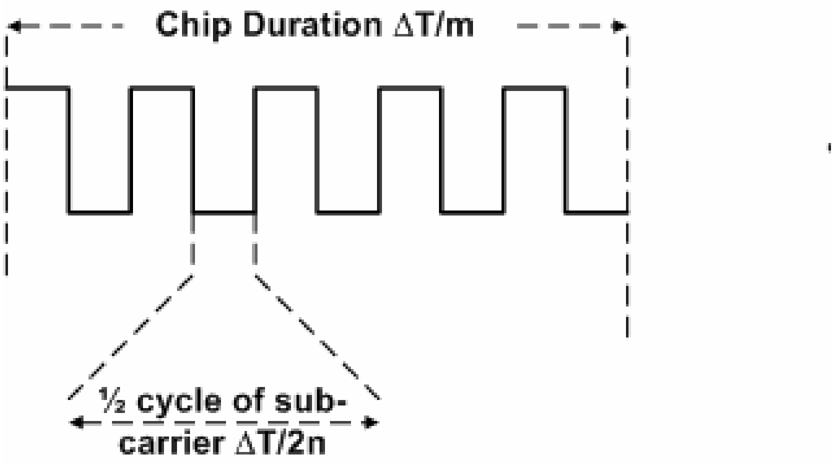

Figure 17.A1

A generic description of the spreading symbol is illustrated in figure 17.A1 and described by the equation below by sub-dividing the spreading symbol into an even number (2n/m) of segments with alternating sign (resulting from the binary sub-carrier):

$$s_T(t) = \begin{cases} (-1)^r & r.\frac{\Delta T}{2n} < t < (r+1).\frac{\Delta T}{2n} \\ 0 & \textit{for values of } t \notin 0..\frac{\Delta T}{m} \end{cases}$$

$$\textit{for integer } r \in 0..(p-1); \textit{and } p = \frac{2n}{m}$$

where ΔT/m is the duration of the code chip and spreading symbol

The Fourier Transform of $s_T(t)$ provides an expression for the (complex) frequency spectrum ($S_T(\omega)$)of the spreading symbol:

$$S_T(\omega) = \int_{-\infty}^{\infty} s_T(t).\exp(-j\omega t).dt \quad \textit{defined on the support } t \in 0..\Delta T/m$$

$$= \sum_{r=0}^{p-1} \left\{ \int_{t=r\Delta T/2n}^{(r+1).\Delta T/2n} (-1)^r.\exp(-j\omega t).dt \right\}$$

$$= \frac{-1}{j\omega} \sum_r (-1)^r . \left\{ \exp\left(-j\omega(r+1)\frac{\Delta T}{2n}\right) - \exp\left(-j\omega r\frac{\Delta T}{2n}\right) \right\}$$

$$= \frac{2j}{j\omega}.\sin\left(\omega.\frac{\Delta T}{4n}\right).\left\{ \sum_r (-1)^r \exp\left(-j\omega(r+\frac{1}{2})\frac{\Delta T}{2n}\right) \right\}$$

The summation can be performed using standard algebraic techniques recognizing a substitution:

$$x = -\exp\left(j\omega\frac{\Delta T}{2n}\right)$$

Then, the summation becomes:

$$x^{\frac{1}{2}} \sum_{r=0}^{p-1} x^r = \frac{1-x^p}{1+x} \cdot x^{\frac{1}{2}}$$

$$= j \cdot x^{\frac{p}{2}} \cdot \frac{\sin\left(\omega\frac{p\Delta T}{4n}\right)}{\cos\left(\omega\frac{\Delta T}{4n}\right)}$$

Using the substitutions above, the complex spectrum of the resulting spreading symbol becomes:

$$S_T(\omega) = 2j \cdot \frac{\Delta T}{4n} \cdot \frac{\sin\left(\omega \frac{\Delta T}{4n}\right)}{\left(\omega \frac{\Delta T}{4n}\right)} \cdot \frac{\sin\left(\omega \frac{p\Delta T}{4n}\right)}{\cos\left(\omega \frac{\Delta T}{4n}\right)} \cdot x^{\frac{p}{2}}$$

The PSD for the spreading symbol with a random ideal code sequence becomes:

$$\Phi_{S(n,m)}(\omega) = \frac{4\Delta T}{m} \cdot \left\{ \frac{\sin\left(\omega \frac{\Delta T}{4n}\right)}{\left(\omega \frac{p\Delta T}{4n}\right)} \cdot \frac{\sin\left(\omega \frac{p\Delta T}{4n}\right)}{\cos\left(\omega \frac{\Delta T}{4n}\right)} \right\}^2$$

$$\textit{where} \quad p = \frac{2n}{m} \quad \textit{(numer of 1/2 cycles of sub-carrier per code chip)}$$

This equation has a number of potential singularities at the following values of ω, due to zeros in the denominator. These can be evaluated:

$$\omega = 0 \quad \textit{and}$$

$$\omega = \frac{2n}{\Delta T} \cdot \pi(2k-1) \quad \textit{for all integer } k$$

The values of the PSD at these are not singular, however. At ω=0, the PSD has a sin(x)/x function with value equal to 1:

$$\Phi_{S(n,m)}(0) = \frac{4\Delta T}{m} \cdot \left\{ 1 \cdot \frac{0}{p} \right\}^2$$

At the other potential singularities; the values are:

$$\Phi_{S(n,m)}\left(\omega_0(2k-1)\right)=\frac{4\Delta T}{m}\cdot\left\{\frac{1}{\left(\frac{\pi}{2}(2k-1)\right)}\right\}^2$$

where $\omega_0=\frac{4n}{\Delta T}\frac{\pi}{2}$

Identical methodology can be employed to the other three cases for cosine-phased sub-carrier modulation and for p even and odd.